Week Six

Pitfalls in Learning from Asset Prices in Linear Factor Pricing Models

Linear Factor Pricing Models

- Project excess returns on $\Delta \ln x$
  - $R_i - R_f = \alpha_{ix} + \beta_{ix} \Delta \ln x + \varepsilon_{ix}$
  - $E(\varepsilon_{ix}) = E(\Delta \ln x \varepsilon_{ix}) = 0$
- Revised risky asset valuation
  - $R_i - R_f = \beta_{ix}(\Delta \ln x - \lambda_x) + \varepsilon_{ix}$
- N. B.: ε_{ix} processes need not be smooth
    - i.e., ε_{ix} can have (unpriced) discontinuous jumps
- N. B.^{2}: All moments conditional as before
  - I am economizing on notation for a change

Basis Portfolio Measurement in Linear Factor Pricing Models

- Project excess returns on β_{ix} via GLS
  - $R_i - R_f = \beta_{ix}(R_x - R_f) + \eta_{ix}$
  - $R_x - R_f = [\beta_x'\Omega_x^{-1}\beta_x]^{-1}\beta_x'\Omega_x^{-1}(R - \iota R_f)$
    $= \Delta \ln x - \lambda_x + [\beta_x'\Omega_x^{-1}\beta_x]^{-1}\beta_x'\Omega_x^{-1}\varepsilon_x$
  - $\Omega_x = \mathrm{Var}(\varepsilon_x)$
- Risk premiums and CSR coefficients
  - $E(R_x - R_f) = \Delta\mu_x - \lambda_x + [\beta_x'\Omega_{xt}^{-1}\beta_x]^{-1}\beta_x'\Omega_x^{-1}E(\varepsilon_x)$
    $= \Delta\mu_x - \lambda_x$

Statistical Measurement of Basis Portfolio Returns I

- Suppose WLLN applies to ε_{x_N}
  - Sufficient condition: $\lim_{N\to\infty} \xi_{max}(\Omega_{\varepsilon_{xN}}) < \infty$
    - $\Omega_{\varepsilon_{xN}} = Var(\varepsilon_{x_N})$; $\xi_{max}(\Omega_{\varepsilon_{xN}})$ = largest eigenvalue of $\Omega_{\varepsilon_{xN}}$
- Well-diversified basis portfolios I
  - w_{P_N} s. t. $w_{P_N}'\iota = 1$ and $\lim_{N\to\infty} w_{P_N}'w_{P_N} = 0$
  - $R_{P_N} - R_f = w_{P_N}'(R_N - \iota_N R_f) = \beta_{P_N x}'(\Delta\ln x - \lambda_x) + w_{P_N}'\varepsilon_{x_N}$
    - $\sigma_{\varepsilon_{pxN}}^2 = Var(\varepsilon_{Px_N}) = Var(w_{P_N}'\varepsilon_{x_N}) = w_{P_N}'\Omega_{\varepsilon_{xN}} w_{P_N}$
  - $\lim_{N\to\infty}\sigma_{\varepsilon_{pxN}}^2 = \lim_{N\to\infty} w_{P_N}'\Omega_{\varepsilon_{xN}} w_{P_N}$
    $\leq \lim_{N\to\infty} w_{P_N}'w_{P_N}\, \xi_{max}(\Omega_{\varepsilon_{xN}}) = 0$
    - Perfect basis portfolios in the limit

Statistical Measurement of Basis Portfolio Returns II

- State variables only source of return *predictability* in state-variable-driven models
  - i.e., $E[R_i - R_f|\mathscr{F}] = \beta_{ix}'[E(\Delta \ln x|\mathscr{F}) - \lambda_x]$; $E[\varepsilon_{ix}|\mathscr{F}] = 0$
  - $\Rightarrow$ choose factors for ability to *predict* returns
  - No need for diversifiable residuals
- State variables both dominant source of covariation among returns and sole source of return predictability in *factor* models
  - Covariance-based measures better on statistical grounds due to greater precision of estimation

Issues in the Identification of State Variables

- State variables in linear SDF model
  - *The* indicator of pricing relevant state
  - Their growth rates determine the SDF
  - Represent sources of uncertainty impinging on investor decisions in this market
- Two generic types of state variables
  - Basis or maximum correlation portfolios
    - *Proximate* causes or determinants R_x
  - Variables measuring risks themselves
    - *Ultimate* causes or determinants x like aggregate macro state variables

Proximate Causes Used in Empirical Research

- Equity portfolio returns
  - S&P 500 and CRSP value-weighted indices
    - large part of traded U. S. stock market wealth
  - Equally-weighted CRSP indices
    - tilted toward small firms with greater exposure to default and financial risks over business cycle
- Default-free and risky bonds
  - Short and long term government bonds
  - Risky bonds rated Baa and below by S&P
- Oil price changes and exchange rates

Prespecified Basis Portfolios in Linear Factor Pricing Models

- Prespecified basis portfolio returns R_{xP}
  - $R_i - R_f = \beta_{ixP}'(R_{xP} - \iota R_f) + \varepsilon_{ixP}$
  - $\Delta \ln x = \phi_{xP} + \Phi_{xP}(R_{xP} - \iota R_f) + \upsilon_{xP}$; $E[\upsilon_{xP}] = 0$
  - When is this relation valid?
    - When $E[\varepsilon_{ixP}\upsilon_{xP}] = 0 \ \forall \ i$
    - i.e., υ_{xP} is news on state variables *not* in prices
- Structural vs. reduced form parameters
  - $\beta_{ixP}' = \beta_{ix}'\Phi_{xP}$ and $\phi_{xP} = \lambda_x$; Φ_{xP} has full row rank
  - *A priori* restrictions on β_{ix} and λ_x of no value without *a priori* information on Φ_{xP} and ϕ_{xP}

Prespecified Basis Portfolios and the Role of the Cross-section

- More assets not needed if maximum correlation portfolios measured *a priori*
  - Many researchers try for reasons beyond me
- Reason 1: expected return measurement
- Reason 2: testing the null model
  - i.e., search for deviations from linearity
- Reason 3: rejection of null $\Rightarrow$ identification of omitted risk factors if linearized state variable pricing model null maintained
  - Discussed below

The Basic Tradeoff in State Variable Measurement

- Models with basis portfolio returns
  - Potentially much less measurement error since returns are measured with great accuracy
  - Not economic objects of interest
- Models with measured state variables
  - Scientific curiosity about ultimate sources of risk/return tradeoffs
    - Role of output or consumption growth, wage or price inflation, permanent, transitory or unexpected components, changes in expectations, and so on
  - Severe potential for measurement error

Classical Measurement in State Variable Proxies

- Classical measurement error is pure noise caused by measurement process
  - i.e., uncorrelated with asset returns
- Error-ridden proxy $\Delta\ln y$
  - $R_i - R_f = \beta_{ixy}'(\Delta\ln y - \lambda_y) + \varepsilon_{ixy}$
  - $\Delta\ln x = \phi_{xy} + \Phi_{xy}\Delta\ln y + \upsilon_{xy}$; $E[\upsilon_{xy}]=E[\upsilon_{xy}\varepsilon_{ixy}]=0$
- Structural vs. reduced form parameters
  - $\beta_{ixy}' = \beta_{ix}'\Phi_{xy}$ and $\phi_{xy} = \lambda_y$; Φ_{xy} has full row rank
  - *A priori* restrictions on β_{ix} and λ_x of no value without *a priori* information on Φ_{xy} and ϕ_{xy}

Conceptual Measurement Errors in State Variable Proxies

- Examples
  - permanent vs. transitory components
  - expected vs. unexpected components
- Can be like classical measurement error
  - Nominal interest rates and *ex post* inflation span innovations in both expected and unexpected inflation
  - Changes in national income span changes in permanent income if transitory income is classical measurement error

Nonclassical Measurement Errors in Macro Variables

- Much measurement error not classical
- Example: measurement errors in national income and product accounts
  - Social and corporate accounting similar
  - Assets valued at book values, i.e, historical costs
  - $\Rightarrow$ Income, wealth, and capital mismeasured in ways correlated with asset returns
  - Consumer durables that create consumption service flows over time yield similar problems
  - $\Rightarrow$ Change in value correlated with asset returns

Implications for Linear Factor Pricing Models

- Error-ridden proxy $\Delta\ln y$ with $E[\upsilon_{xy}\varepsilon_{ixy}] \neq 0$
  - $R_i - R_f = \alpha_{iy} + \beta_{iy}'(\Delta\ln y - \lambda_y) + \varepsilon_{ixy}$
  - $\Delta\ln x = \phi_{xy} + \Phi_{xy}\Delta\ln y + \upsilon_{xy}$; $E[\upsilon_{xy}] = 0$
  - $\lambda_y = R_f \, \mathrm{Cov}(\Delta\ln y, m)$
  - $\alpha_{ixy} = -R_f \, \mathrm{Cov}(\varepsilon_{ixy}, m)$
    - Difficult to bound $\mathrm{Cov}(\varepsilon_{ixy}, m)$ *a priori*
- Obvious role for models
  - Imputation of associated capitalized values
  - Estimation of relevant decompositions
  - Models of service flows

Proximate Vs. Ultimate Causes Once Again

- Empirically successful models rely on asset returns as factors
- Natural progression seems to be to:
  - first model asset pricing relations in terms of their proximate causes and
  - then investigate relations between measured proximate and potential ultimate causes
- Alternatively begin with a more explicit, equilibrium model that links theoretical constructs to observed state variables

Learning From Misspecified Linear Factor Pricing Models

- Begin with the false model
  - $R_i - R_f = \alpha_{iy} + \beta_{iy}'(\Delta \ln y - \lambda_y) + \varepsilon_{iy}$
- Project $\mathcal{R}_i - \mathcal{R}_f$ on $B_y = \{\beta_{iy}\}$ via GLS
  - $\mathcal{R}_i - \mathcal{R}_f = \beta_{iy}'(\mathcal{R}_y - \imath\mathcal{R}_f) + \alpha_{iy}$; $B_y'\Omega_{\varepsilon_y}^{-1}\alpha_y = 0$
- Construct new basis portfolio return
  - $R_{\alpha_y} - R_f = (\alpha_y'\Omega_{\varepsilon_y}^{-1}\alpha_y)^{-1}\alpha_y'\Omega_{\varepsilon_y}^{-1}(R - \imath R_f)$
  - $w_{\alpha_y} = \Omega_{\varepsilon_y}^{-1}\alpha_y(\alpha_y'\Omega_{\varepsilon_y}^{-1}\alpha_y)^{-1}$
  - $E(R_{\alpha_y} - R_f) = 1$
  - $Var(R_{\alpha_y} - R_f) = (\alpha_y'\Omega_{\varepsilon_y}^{-1}\alpha_y)^{-1}$

Reverse Engineering 'True' Linear Factor Pricing Models

- Asset pricing model based on $\{R_{\alpha_y}, \Delta \ln y\}$
  - $R_i - R_f = \delta_i + \beta_{i\alpha_y} R_{\alpha_y} + \beta_{iy}'(\Delta \ln y - \lambda_y) + \varepsilon_{i\alpha_y}$
  - $\beta_{\alpha_y} = \mathrm{Cov}[R, R_{\alpha_y}]/\mathrm{Var}(R_{\alpha_y})$
    $= E(\varepsilon_{\alpha_y}\varepsilon_{\alpha_y}'\Omega_{\varepsilon_y}^{-1})\alpha_y(\alpha_y'\Omega_{\varepsilon_y}^{-1}\alpha_y)^{-1}/(\alpha_y'\Omega_{\varepsilon_y}^{-1}\alpha_y)^{-1}$
    $= \alpha_y$
  - $\Rightarrow R_i - R_f = \alpha_{iy} R_{\alpha_y} + \beta_{iy}'(\Delta \ln y - \lambda \alpha_y) + \varepsilon_{i\alpha_y}$; $E(\varepsilon_{i\alpha_y}) = 0$
- What is insidious about this model
  - Purely mechanical revision of false model
  - Data mining, not economics
    - Only economics is absence of arbitrage

The Interpretation of Reverse Engineered Models

- It is *always* possible to use α_y to construct mean-variance efficient portfolios
- It is hard to call reverse engineered model successful reflection of any economic intuition underlying selection of $\Delta \ln y$ with no theoretical justification for portfolio α_y
- Any such claim involves an ancillary hypothesis about unobservables:
  - R_{α_y} represents an omitted risk factor

The Omitted Risk Factor Interpretation

- Mechanical artifact of linear regression
  - Projection of m onto $\Delta \ln y$ and excess asset returns equals projection onto $\Delta \ln x$ and $R_{\alpha_y} - R_f$
- m represents risk on this market
  - $\Rightarrow$ no abuse of language to refer to $R_{\alpha_y} - R_f$ as omitted risk factor
    - perhaps portfolio of omitted risk factors
- $\Rightarrow$ View successful multifactor model as reduced form for theory to explain
  - Model silent on structural risk/return model